Dynamic Matching Algorithms for Homelessness Reduction

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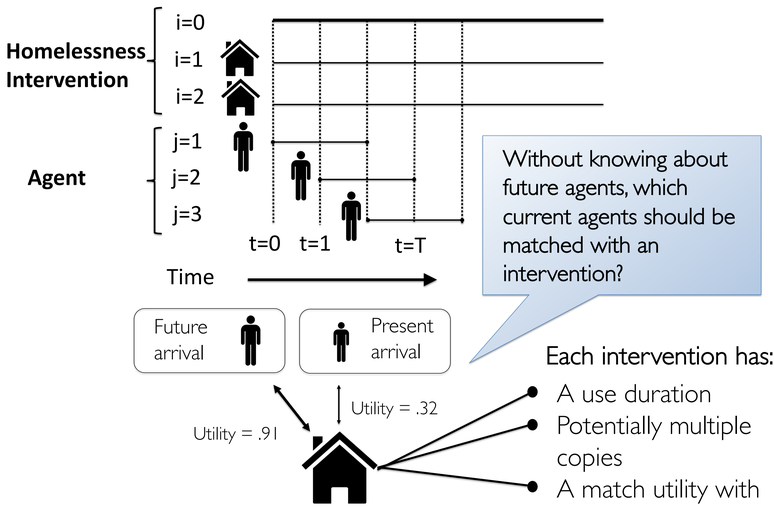
Introduction

Finding optimal homelessness intervention allocations is complicated by dynamic arrivals and departures of homeless agents[1]. Primal-dual techniques have been used in online allocation contexts previously, and could address this problem[2].

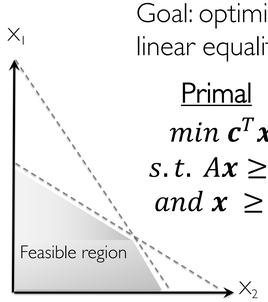
This work aims to develop an online primal-dual based dynamic matching strategy for pairing homeless agents with interventions.

The Housing Assignment Problem



Linear Programming

Goal: optimize a linear objective function subject to linear equality and inequality constraints.



Primal

$$min\ \boldsymbol{c}^T\boldsymbol{x}$$
$$s.t.\ A\boldsymbol{x} \geq \boldsymbol{b}$$
$$and\ \boldsymbol{x} \geq 0$$

Any primal lp has an equivalent dual

Dual

$$max\ \boldsymbol{b}^T\boldsymbol{y}$$
$$s.t.\ A^T\boldsymbol{y} \leq \boldsymbol{c}$$
$$and\ \boldsymbol{y} \geq 0$$

Where $\boldsymbol{x}$ is a decision variable, and $\boldsymbol{y}$ is a linear combination of primal constraints (dual variable).

Problem Formulation

Primal Formulation

$$max \sum_{i=0}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} x_{i,j,t} w_{i,j,t}$$

$$s.t.$$

$$\sum_{t'=0}^{k_i-1}\sum_{j=0}^{J} x_{i,j,t+t'} \leq c_i, \forall i,t$$

$$\sum_{i=1}^{I}\sum_{t=0}^{T} x_{i,j,t} \leq 1, \forall j$$

$$x_{i,j,t} \geq 0, \forall i,j,t$$

$x_{i,j,t} \in \{0,1\}$. Binary match variable

$w_{i,j,t} \in \Re$. Match quality

Dual: Resource use over time

Dual Formulation

$$min \sum_{i=0}^{I}\sum_{t=1}^{T} c_i\alpha_{i,t} + \sum_{j=1}^{J}\beta_j$$

$$s.t.$$

$$w_{i,j,t} - \sum_{t'=t}^{min(t+k_i-1,T)} \alpha_{i,t'} - \beta_j \leq 0, \forall i,j,t$$

$$\alpha_{i,t}, \beta_j \geq 0, \forall i,j,t$$

Dual: Agent quality

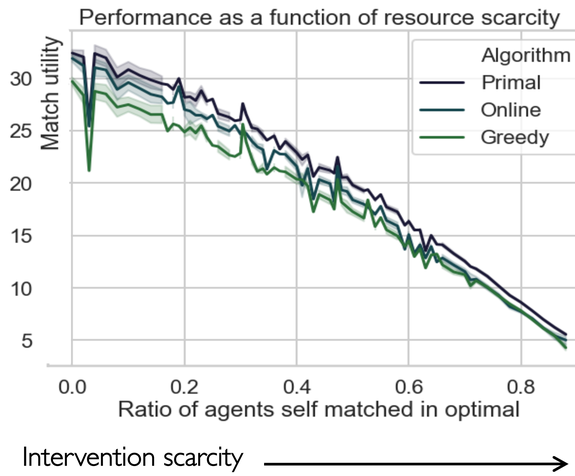
- Directly solving the primal is infeasible since we don't know future arrivals.
- Using strong duality and complementary slackness, if we are given an optimal dual, we can devise an online assignment algorithm.

Online Dual-Based Assignment

For each non-matched, available agent in each round, allocate to:

$$i^* = \max_i \{\ w_{i,j,t} - \sum_{t'=t}^{min(t+k_i-1,T)} \alpha_{i,t'} - \beta_j\ \}$$ (if i*=0, don't consider agent matched)

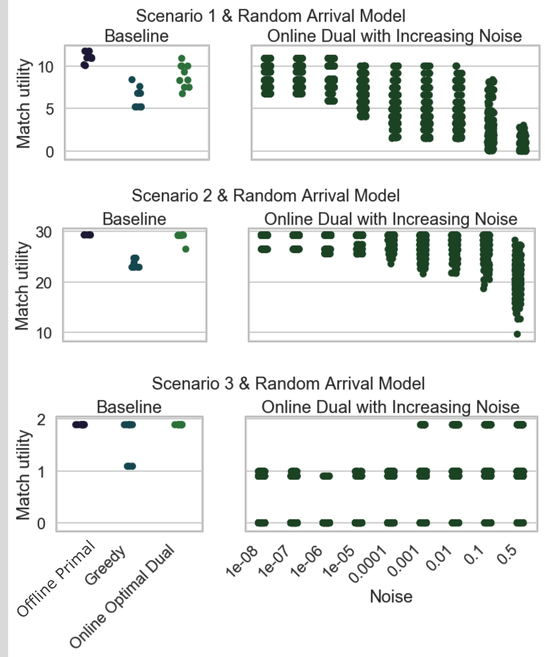
Dual-based vs. Greedy Matching



Online dual-based (with optimal dual) out-performs greedy matching, especially when resources are abundant..

In practice, it is unlikely to obtain an optimal dual. We next investigate how dual noise deteriorates performance.

Stress Tests & Failure Cases



Scenario 1

	Agent A	Agent B
Resource X	.05	.05
Resource Y	.1	.9

$Scarcity_{Res\ X} \ll Scarcity_{Res\ Y}$

Scenario 2

	Agent A	Agent B
Resource X	.99	.99
Resource Y	.05	.95

$Scarcity_{Res\ X} == Scarcity_{Res\ Y}$

Scenario 3

	Agent A	Agent B
Resource X	.05	.05
Resource Y	.05	.95

1 copy of resource X & Y

Discussion

Online dual-based matching yields a more-optimal pairing of homeless agents compared with greedy matching. This is particularly pronounced in situations where greedy assignment fails to consider future information and dual noise is limited.

Next steps will investigate simulation and machine learning methods for learning dual variables without oracle knowledge.

References

1. Kube, A., Das, S., & Fowler, P. J. (in press). Allocating interventions based on counterfactual predictions: A case study on homelessness services. *In Proceedings of the Association for the Advancement of Artificial Intelligence* (AAAI)
2. Li, Z., Lieberman, K., Macke, W., Carrillo, S., Ho, C., Wellen, J., & Das, S. (2019). Incorporating Compatible Pairs in Kidney Exchange. Proceedings of the 2019 ACM Conference on Economics and Computation - EC 19. doi:10.1145/3328526.3329619

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